A Unified Framework for Symmetry Breaking in SO(10)

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Abstract

A new SO(10) unified model is proposed based on a one step breaking of SO(10) to the Standard Model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ using a single 144 of Higgs. The symmetry breaking occurs when the SU(5) 24-plet component of 144 develops a vacuum expectation value. Further, it is possible to obtain from the same 144 a light Higgs doublet necessary for electro-weak symmetry breaking using recent ideas of string vacua landscapes and fine tuning. Thus the breaking of SO(10) down to $SU(3)_C \times U(1)_{em}$ can be accomplished with a single Higgs. We analyze this symmetry breaking pattern in the nonsupersymmetric as well as in the supersymmetric SO(10) model. In this scenario masses of the quarks and leptons arise via quartic couplings. We show that the resulting mass pattern is consistent with experimental data, including neutrino oscillations. The model represents an alternative to the currently popular grand unified scenarios.

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1 Introduction

In any Grand Unified Theory (GUT) understanding the Higgs sector is not an easy task. Usually these models require the existence of more than one Higgs multiplet. In the minimal $SU(5)$ GUT one employs one adjoint 24 -plet (Σ) and a fundamental 5-plet to break the GUT symmetry down to $SU(3)_C \times U(1)_{em}$. The Yukawa couplings of the 5-plet Higgs also generate quark and lepton masses. The Higgs sector becomes somewhat more complicated in larger GUT structures such as $SO(10)$[1] since there is a larger symmetry that needs to be broken. Conventional $SO(10)$ models employ at least two different Higgs representations to break the symmetry down to $SU(3)_C \times SU(2)_L \times U(1)_Y$ (a 16 or a 126 to change rank, and one of 45, 54 or a 210 to break the symmetry down further [2]). To achieve electro-weak symmetry breaking and to generate quark and lepton masses an additional 10 dimensional representation is also needed. A minimal $SO(10)$ model studied recently, for example, utilizes one 10, one 126 and one 210 Higgs representations to achieve symmetry breaking and to generate masses for the quarks, leptons and the neutrinos[3].

In this paper we discuss the following question: Is it possible to achieve $SO(10)$ symmetry breaking all the way down to the $SU(3)_C \times U(1)_{em}$ with a single Higgs representation? We find that this is indeed the case if one employs a 144-plet of Higgs of $SO(10)$. The 144-plet is contained in the product 10×16, and thus carries one vector and one spinor index. An interesting property of the 144-plet which makes such a symmetry breaking chain possible is that it contains in it an $SU(5)$ adjoint with a $U(1)$ charge, as well as Standard Model Higgs doublet fields. This can be seen from the following decomposition of 144 under $SU(5) \times U(1)$ subgroup of $SO(10)$

$$144 = \bar{5}(3) + 5(7) + 10(-1) + 15(-1) + 24(-5) + 40(-1) + \overline{45}(3) \tag{1}$$

It is significant that the $SU(5)$ adjoint 24(-5) above also carries a U(1) charge. Once the Standard Model singlet in it acquires a VEV, it would change the rank of the group, leading to a one-step breaking of $SO(10)$ down to $SU(3)_C \times SU(2)_L \times U(1)_Y$. The sub-multiplets $\bar{5}(3), 5(7)$ and $\overline{45}(3)$ all contain fields with identical quantum numbers as the Standard Model Higgs doublet. If one combination of doublets from these sub-multiplets is made light by fine tuning, it can be used for electro-weak symmetry breaking. Such fine tuning is justified in the context of the multiple vacua of the string landscapes, which has been widely discussed recently. Although consistency of the first stage of symmetry breaking requires the mass-squared of all the physical Higgs particles to be positive (including that of the light Higgs doublet), we show that radiative corrections involving the Higgs self-couplings can turn the

mass-squared of the light Higgs doublet negative, facilitating the second stage of symmetry breaking.

Realistic fermion masses can be obtained within this minimal scenario. Recall that the fermions of each family belongs to the 16 dimensional spinor representation of $SO(10)$. Under $SU(5) \times U(1)$ subgroup the 16 decomposes as follows

$$16 = 10(-1) + \bar{5}(3) + 1(-5) \tag{2}$$

Fermion masses will arise from quartic couplings of the $16_i 16_j (144\,144)$ and $16_i 16_j\ (144^*\,144^*)$. These couplings would lead to Dirac masses for all the fermions as well as large Majorana masses for the right-handed neutrinos. Since the light Higgs doublet is a linear combination of doublets from 5 and 45 of $SU(5)$, the resulting mass pattern is not that of minimal $SU(5)$ and is consistent with experimental data, including neutrino oscillations.

The outline of the rest of the paper is as follows: In Sec. 2 we discuss symmetry and mass growth in the $SU(5) \times U(1)$ language. In Sec. 3 we discuss the techniques of calculation for the analysis of 144 and $\overline{144}$ plet couplings using the method developed in Ref.[4]. Here also we discuss the set of couplings $(144\times\overline{144})$, $(144\times\overline{144})_1(144\times\overline{144})_1$, $(144\times\overline{144})_{45}(144\times\overline{144})_{45}$ and $(144\times\overline{144})_{210}(144\times\overline{144})_{210}$. These couplings are needed in the computation of symmetry breaking which is then analysed. In Sec. 4 Higgs phenomenon and mass growth are analysed for the breaking of $SO(10)$. Here it is shown that within the landscape scenario with fine tuning[9, 10] one gets a pair of light Higgs doublets exactly as in the minimal supersymmetric standard model (MSSM) while the Higgs triplets and other modes are either absorbed or become super heavy. In Sec. 5 couplings of quarks and leptons are discussed and it is shown that such couplings are quartic in nature. As an illustration the couplings involving $(16 \times 16)_{10}(144 \times 144)_{10}$ and $(16 \times 16)_{10}(\overline{144} \times \overline{144})_{10}$ are explicitly discussed. It is shown that the resulting masses and mixings are consistent with experimental data. Conclusions are given in Sec. 6.

2 Symmetry breaking and mass growth with 144 in the $\mathbf{SU(5) \times U(1)}$ language

Analysis of the symmetry breaking and of fermion mass generation with a 144 of Higgs turns out to be rather complicated. Before we delve into the full detail in the $SO(10)$ language, which is presented in the next section, we analyze here these issues in the simpler $SU(5) \times U(1)$ subgroup language. We will present our analysis in a non-supersymmetric

model. Generalization to supersymmetry require the addition of a $\overline{144}$ chiral multiplet, so that the flatness of the D-term potential can be maintained at the GUT scale, leaving supersymmetry intact at that scale. The analysis in this section would also hold for SUSY models with some redefinitions of parameters, provided that the 144^* field is identified with the $\overline{144}$ of the SUSY $SO(10)$ model.

2.1 One step GUT symmetry breaking

Since in the $SU(5) \times U(1)$ decomposition of $SO(10)$ the 144 contains an $SU(5)$ adjoint carrying a non-zero $U(1)$ charge (see Eq.(1)), it is instructive to analyze symmetry breaking of $SU(5) \times U(1)$ with a complex adjoint Σ. One can construct such a representation from two adjoint representations of $SU(5)$: $\Sigma = \Sigma_1 + i\Sigma_2$. Then Σ is not self-adjoint, and we denote the conjugate of Σ as $\Sigma^\dagger$.

The most general $SU(5) \times U(1)$ invariant renormalizable potential involving the Σ and $\Sigma^\dagger$ fields is

$$\begin{aligned} V = -M^2 tr(\Sigma\Sigma^\dagger) + \frac{\kappa_1}{2} tr(\Sigma^2\Sigma^{\dagger 2}) + \frac{\kappa_2}{2}(tr(\Sigma\Sigma^\dagger))^2 \\ + \frac{\kappa_3}{2} tr(\Sigma^2) tr(\Sigma^{\dagger 2}) + \frac{\kappa_4}{2} tr(\Sigma\Sigma^\dagger\Sigma\Sigma^\dagger) \ . \end{aligned} \tag{3}$$

Among the possible local minima is the one which preserves the Standard Model gauge symmetry given by the vacuum structure

$$< \Sigma > = < \Sigma^\dagger > = v \ diag(2, 2, 2, -3, -3) \tag{4}$$

For some range of the parameters of the potential, this minimum will be the global minimum. Minimization of the potential gives

$$v^2 = \frac{M^2}{7(\kappa_1 + \kappa_4) + 30(\kappa_2 + \kappa_3)} \ . \tag{5}$$

Clearly this VEV structure breaks $SU(5)$ down to $SU(3)_C \times SU(2)_L \times U(1)_Y$. Further, since Σ is charged under the $U(1)$, its VEV breaks this $U(1)$. Thus we see that the $SU(5) \times U(1)$ symmetry is broken down to the SM gauge symmetry in one step with one complex adjoint Higgs field. This can also be verified directly by computing the gauge boson masses. The physical Higgs boson masses can all be made positive for some range of parameters of the potential. It is then clear that if an $SO(10)$ representation contains sub-multiplets which transform under $SU(5) \times U(1)$ symmetry as an adjoint carrying $U(1)$ charge, then there is the possibility that this Higgs field can break $SO(10)$ all the way down to the SM gauge symmetry

in one step. We observe that the 144-plet of $SO(10)$ is the simplest representation which has this property[2]. Technical details of this assertion in the $SO(10)$ language is postponed to the next section.

Identical conclusions can be arrived at for the case of supersymmetric $SU(5) \times U(1)$ model. The potential of Eq.(3) will become the superpotential (with Σ^* replaced by a chiral superfield $\overline{\Sigma}$), the couplings κ_i will be nonrenormalizable operators with inverse dimensions of mass, and the mass term M^2 will be replaced by M. Thus we conclude that in SUSY $SO(10)$ a $144 + \overline{144}$ pair of chiral superfields can break $SO(10)$ in one step down to the supersymmetric Standard Model gauge group.

2.2 Electroweak symmetry breaking

Having recognized that a single 144-plet can break non-supersymmetric $SO(10)$ down to the SM in one step, we turn our attention to the subsequent electro-weak symmetry breaking. As noted earlier, the 144-plet also contains fields which have the same quantum numbers as the SM Higgs doublet. We explain how these doublet fragments from the 144-plet can be used for the purpose of electro-weak symmetry breaking, thus making the model very economical.

An immediate question that can be raised is how to obtain negative mass-squared for the light Higgs doublet of the SM arising from the 144-plet. Consistency of the GUT symmetry breaking would require positivity of the mass-squared of all the physical Higgs bosons, including that of the light SM doublet. If the surviving symmetry and spectrum below the GUT scale are corresponding to those of the SM, there would be no interactions that turn the Higgs mass-squared negative needed for electroweak symmetry breaking. In analogy to the stop squark quartic couplings turning the Higgs mass-squared negative in the supersymmetric SM, we observe that the quartic couplings between the light Higgs doublet and any fragment of the 144-plet with mass an order of magnitude or so below the GUT scale can turn the Higgs doublet mass-squared negative[3]. We illustrate this mechanism with a simple $SU(5)$ toy model with an adjoint Higgs field below.

Consider a toy model with global $SU(5)$ symmetry broken spontaneously by an adjoint

[2] The next simplest possibility is to have a 560 of $SO(10)$, which contains a 24(-5), 1(-5), as well as $\bar{5}(3), 45(7)$, and $\overline{45}(3)$ under $SU(5) \times U(1)$.

[3] Yukawa couplings between the Higgs doublet and fermions cannot turn the Higgs mass-squared negative. Cubic self couplings (which are not allowed in the SM Higgs doublet) and/or quartic/cubic scalar couplings involving other fields are necessary for this to happen.

Higgs field Σ. The most general renormalizable potential for this field is given by

$$V = m^2 Tr(\Sigma^2) + \frac{\kappa_1}{2} tr(\Sigma^4) + \frac{\kappa_2}{2} (tr(\Sigma^2))^2 + \mu tr(\Sigma^3). \tag{6}$$

One possible VEV structure is as shown in Eq.(4). Minimization of the potential of Eq.(6) gives

$$m^2 = -(7\kappa_1 + 30\kappa_2)v^2 + \frac{3}{2}\mu v \ . \tag{7}$$

The $(3, 2, -5/6)$ and $(3^*, 2, 5/6)$ components of Σ (under the surviving $SU(3)\times SU(2)\times U(1)$ symmetry) are Goldstone bosons, while the $(8, 1, 0)$, $(1, 3, 0)$ and the $(1, 1, 0)$ components have masses given respectively by

$$\begin{aligned} m_8^2 &= \frac{15}{2}\mu v + 5\kappa_1 v^2, \\ m_3^2 &= -\frac{15}{2}\mu v + 20\kappa_1 v^2, \\ m_1^2 &= -\frac{3}{2}\mu v + (14\kappa_1 + 60\kappa_2)v^2. \end{aligned} \tag{8}$$

For a range of parameters, all three squared-masses can be chosen positive, establishing the consistency of symmetry breaking.

It is possible by fine tuning to make the $SU(2)_L$ triplet field to be much lighter than the $SU(5)$ breaking scale, while keeping the other two components heavy. We wish to ask if such a finely tuned triplet can subsequently break $SU(2)_L$ further down to $U(1)_L$. This would, however, require that m_3^2 turn negative at lower scales even though it starts off as being positive at the high scale. Consider the case when κ_1 and μ/v are somewhat smaller than one (say of order 0.01), while κ_2 is of order one. Then m_3 and m_8 are generically an order of magnitude below the GUT scale v, while m_1 is of order v. In the momentum range below v and above m_8, the mass parameters m_3^2 and m_8^2 will evolve, while the singlet decouples. The RGE for the running of these mass parameters are found to be

$$\begin{aligned} \frac{dm_8^2}{dt} &= \frac{1}{8\pi^2}[\frac{15}{8}(\mu + 4\kappa_1 v)^2 + \frac{5}{2}(\kappa_1 + 2\kappa_2)m_8^2 + \frac{3}{2}\kappa_2 m_3^2] \\ \frac{dm_3^2}{dt} &= \frac{1}{8\pi^2}[4\kappa_2 m_8^2 + (\frac{1}{2}\kappa_1 + \kappa_2)m_3^2] \end{aligned} \tag{9}$$

where $t = log(Q)$. With κ_2 being of order one and κ_1, $\mu/v << 1$, we see from these equations that if m_3^2 at the scale v starts off being smaller than m_8^2, it can turn negative in going down from v to the mass scale m_8. The mass parameter m_8^2 will remain positive in this case. This

example shows that fine tuning of the weak triplet can be done at the scale m_8 is such a way that its squared-mass turns negative at lower energy scales.

In analogy with this example, if any fragment of the 144-plet of $SO(10)$ remains somewhat lighter than the GUT scale, then the quartic couplings involving that fragment and the Higgs doublet would turn the doublet mass-squared negative[4]. It is interesting to note that if such fragments from the 144 that survive below the GUT scale are the color octet(s) and the weak triplet(s), unification of the three SM gauge couplings will occur nicely at a scale of $(10^{16}-10^{17})$ GeV [5]. The above analysis and conclusions are in the context of a non-SUSY $SO(10)$ scenario. In the SUSY $SO(10)$ case the top-stop Yukawa couplings will turn the Higgs doublet mass-square negative in the usual way.

2.3 Doublet-triplet splitting

We denote the components of the 144 multiplet by Q's. As can be seen from Eq.(1) the Higgs fields reside in the multiplets $Q_i(\bar{5})+Q^i(5)+Q^i_j(24)+Q^k_{ij}(\overline{45})$. Similarly we denote the components of the 144* multiplet by P's and in this case the relevant Higgs multiplets will be $P_i(\bar{5})+P^i(5)+P^i_j(24)+P^{ij}_k(45)$. The $\overline{5}, 5$ and 45 representations contain $SU(2)_L$ doublets and $SU(3)_C$ triplets. Before studying the doublet-triplet splitting in the full $SO(10)$ theory, here we analyze this possibility within the simpler $SU(5)\times U(1)$ theory, but with couplings motivated by the full $SO(10)$ theory. The relevant potential that we consider is given by

$$\begin{aligned}V_{DT} = {} & m^2 tr(Q_iP^i)+m^2tr(Q^iP_i)+m^2tr(Q^k_{ij}P^{ij}_k)+\lambda_1 tr(Q_iQ^i)tr(Q^i_jQ^j_i)\\ & +\lambda_2 tr(P_iP^i)tr(P^i_jP^j_i)+\lambda_3 tr(Q_iP^i)tr(Q^i_jP^j_i)+\lambda'_3 tr(Q^iP_i)tr(Q^i_jP^j_i)\\ & +\lambda_4 tr(Q^k_{ij}P^{ij}_k)tr(Q^i_jP^j_i)+\eta_1 tr(Q_iQ^iQ^j_kQ^k_j)+\eta_2 tr(P_iP^iP^j_kP^k_j)\\ & +\eta_3 tr(Q_iP^iQ^j_kP^k_j)+\eta'_3 tr(Q^iP_iQ^j_kP^k_j)+\eta_4 tr(Q^k_{ij}P^mQ^i_kP^j_m)+\eta'_4 tr(P^{ij}_kP_mP^k_iP^m_j)\\ & +\eta_5 tr(Q^k_{ij}Q^mQ^i_kQ^j_m)+\eta'_5 tr(P^{ij}Q_mP^i_kQ^j_m)+\eta_6 tr(P^{ij}_kQ^k_{lm}Q^l_iP^m_j)\end{aligned} \tag{10}$$

where λ_i and η_i are dimensionless couplings which represent different types of contractions of indices. It is easily checked that each term in the potential has an overall zero $U(1)$ quantum number. There are several Higgs doublets and Higgs triplets and anti-triplets in this model. Thus one set of Higgs doublets and triplets and anti-triplets arise from Q_i, Q^i, P_i, P^i. Specifically the doublets are $Q_\alpha, Q^\alpha, P_\alpha, P^\alpha$, while the triplets (anti-triplets)

[4]This statement doas not contradicts Michel-Radicati theorem[6] as we are using radiative correction to turn the doublet mass-squared negative.

are $Q^a, P^a(Q_a, P_a)$. Additional Higgs doublets, triplets and anti-triplets arise from the 45 of $SU(5)$ (from the $\overline{144}$-plet) and from the $\overline{45}$ (from the 144-plet). We discuss the decomposition of these below. The 45 of $SU(5)$ embedded in $\overline{144}$ has the following $SU(2)\times SU(3)\times U(1)_Y$ decomposition

$$\begin{aligned}45 = (2,1)(3) + (1,3)(-2) + (3,3)(-2) + (1,\bar{3})(8)\\ +(2,\bar{3})(-7) + (1,\bar{6})(-2) + (2,8)(3)\end{aligned} \tag{11}$$

One notices that while there is only one $SU(2)$ Higgs doublet ($\tilde{P}^\alpha, \alpha = 1,2$), there is one $SU(3)_C$ Higgs triplet $\tilde{P}^a$ ($a = 1,2,3$) and one anti-triplet $\tilde{P}_a$. Similarly, for the $\overline{45}$ embedded in 144 one has one $SU(2)$ Higgs doublet ($\tilde{Q}_\alpha, \alpha = 1,2$), one $SU(3)_C$ Higgs triplet $\tilde{Q}^a$ (a=1,2,3) and one anti-triplet $\tilde{Q}_a$. The above analysis shows that the Higgs doublet mass matrix will be 3×3 while the Higgs triplet mass matrix will be 4×4. We focus first on the Higgs doublet mass matrix after Q^i_j and P^i_j develop VEVs. We display the mass matrix for the Higgs doublets in the basis where the rows are $(P_\alpha, Q_\alpha, h_\alpha)$ and the columns by $(P^\alpha, Q^\alpha, h^\alpha)$

$$\begin{bmatrix} 30\lambda_2 v^2 + 9\eta_2 v^2 & m^2 + 30\lambda'_3 v^2 + 9\eta'_3 v^2 & v^2\eta'_4 c \\ m^2 + 30\lambda_3 v^2 + 9\eta_3 v^2 & 30\lambda_1 v^2 + 9\eta_1 v^2 & v^2\eta'_5 c \\ v^2\eta_4 c & v^2\eta_5 c & m^2 + 30\lambda_4 v^2 + \frac{21}{4}\eta_6 v^2 \end{bmatrix} \tag{12}$$

where $c = -15\sqrt{3}/2\sqrt{2}$. Next, we display the mass matrix for the Higgs triplets in the basis where the rows are labelled $(P_a, Q_a, \tilde{Q}_a, \tilde{P}_a)$ and the columns by $(P^a, Q^a, \tilde{P}^a, \tilde{Q}^a)$. In this basis the Higgs triplet mass matrix is

$$\begin{bmatrix} 30\lambda_2 v^2 + 4\eta_2 v^2 & m^2 + 30\lambda'_3 v^2 + 4\eta'_3 v^2 & \tilde{c}\eta'_4 v^2 & 0 \\ m^2 + 30\lambda_3 v^2 + 4\eta_3 v^2 & 30\lambda_1 v^2 + 4\eta_1 v^2 & \tilde{c}\eta'_5 v^2 & 0 \\ \tilde{c}\eta_4 v^2 & \tilde{c}\eta_5 v^2 & m^2 + 30\lambda_4 v^2 + \eta_6 v^2 & 0 \\ 0 & 0 & 0 & m^2 + 30\lambda_4 v^2 + 9\eta_6 v^2 \end{bmatrix} \tag{13}$$

where $\tilde{c} = 5\sqrt{2}$. It is easy to see from the determinants of Eqs.(12) and (13) that one can arrange for a pair of light Higgs doublets while keeping the Higgs triplets heavy. As will be shown in Eqs.(50) and (51) from the full $SO(10)$ analysis,we can identify one of the superposition of the doublet fragments as the SM Higgs doublet while keeping the Higgs color triplets fragments all super heavy. It is true that one must fine tune in order to have the Higgs doublet light, but we find it very interesting that with a single 144-plet complete breaking of the $SO(10)$ symmetry down to the residual $SU(3)_C\times U(1)_{em}$ can be achieved.

2.4 Fermion mass growth

For the fermion masses we have the following quartic coupling allowed by gauge invariance, in terms of $SU(5)\times U(1)$ decomposition

$$W=\frac{h_1}{M}10^{ij}10^{kl}\Sigma^j_m Q^m+\frac{h_2}{M}10^{ij}10^{kl}\Sigma^n_l Q^m+\frac{h_3}{M}10^{ij}\bar{5}_i\Sigma^m_j\bar{P}_m+\frac{h_4}{M}10^{ij}\bar{5}_l\Sigma^l_i\bar{P}_j \tag{14}$$

From Eq.(14) we see that it is possible to realize Georgi-Jarlskog type relations with appropriate choice of the h_i couplings even without the 45 of $SU(5)$ acquiring electroweak VEV. In general, there are additional terms involving the 45 of $SU(5)$, which would provide additional freedom since the Higgs doublet will now be a linear combination of 5 and 45.

3 Calculational Techniques for the full SO(10) analysis

In this section we discuss the breaking of $SO(10)$ down to $SU(3)_C\times U(1)_{em}$ in a single step by a single pair of $144+\overline{144}$. Our analysis will be valid for the supersymmetric $SO(10)$ model as well as for the non–supersymmetric model. In the latter case one simply identifies $\overline{144}$ with the 144^* field. However, for formal reasons we consider a single pair of $160+\overline{160}$ where the additional $16+\overline{16}$ that reside in $160+\overline{160}$ are needed for consistency as we will see below. The analysis involving the $144+\overline{144}$ is rather intricate and we use the techniques developed recently in Refs[4] based on the oscillator method of Refs.[7, 8] to compute the desired couplings. There are no cubic couplings of quarks and leptons with the $144+\overline{144}$ of Higgs and one needs quartic couplings to grow quark and lepton masses in this scheme. We discuss now the basic ingredients of the model. We begin by displaying the particle content of $144+\overline{144}$ in multiplets of $SU(5)$. For $\overline{144}$ plet one has

$$\overline{144}=\bar{5}(\mathcal{P}_i)+5(\mathcal{P}^i)+\overline{10}(\mathcal{P}_{ij})+\overline{15}(\mathcal{P}^{(S)}_{ij})+24(\mathcal{P}^i_j)+\overline{40}(\mathcal{P}^l_{ijk})+45(\mathcal{P}^{ij}_k) \tag{15}$$

where the subscripts and superscripts $i,j,k,..$ are $SU(5)$ indices which take on the values $1,..,5$. Similarly, for 144 we find

$$144=5(\mathcal{Q}^i)+\bar{5}(\mathcal{Q}_i)+10(\mathcal{Q}^{ij})+15(\mathcal{Q}^{ij}_{(S)})+24(\mathcal{Q}^i_j)+40(\mathcal{Q}^{ijk}_l)+\overline{45}(\mathcal{Q}^i_{jk}) \tag{16}$$

To make progress we need a field theoretic description of the 144 and $\overline{144}$ plet of fields. Possible candidates are the vector-spinors $|\Psi_{(\pm)\mu}>$. However, an unconstrained vector spinor has $16\times 10=160$ independent components and is reducible. Thus irreducible vector-spinors with dimensionality 144 must be gotten by removing 16 of the 160 components of

the unconstrained vector-spinor. We define the 144-dimensional constrained vector spinors $|\Upsilon_{(\pm)\mu}>$ by imposition of the constraint

$$\Gamma_\mu|\Upsilon_{(\pm)\mu}>=0 \tag{17}$$

where Γ_μ satisfy a rank-10 Clifford algebra

$$\{\Gamma_\mu,\Gamma_\nu\}=2\delta_{\mu\nu}. \tag{18}$$

and where μ,ν are the $SO(10)$ indices and take on the values $1,..,10$.

We discuss now further the relationship of $|\Psi_{(\pm)\mu}>$ and $|\Upsilon_{(\pm)\mu}>$. We begin by writing the 160 and $\overline{160}$ component spinor:

$$|\Psi_{(+)\acute{a}\mu}>=|0>\mathbf{P}_{\acute{a}\mu}+\frac{1}{2}b_i^\dagger b_j^\dagger|0>\mathbf{P}_{\acute{a}\mu}^{ij}+\frac{1}{24}\epsilon^{ijklm}b_j^\dagger b_k^\dagger b_l^\dagger b_m^\dagger|0>\mathbf{P}_{\acute{a}i\mu} \tag{19}$$

$$|\Psi_{(-)\acute{b}\mu}>=b_1^\dagger b_2^\dagger b_3^\dagger b_4^\dagger b_5^\dagger|0>\mathbf{Q}_{\acute{b}\mu}+\frac{1}{12}\epsilon^{ijklm}b_k^\dagger b_l^\dagger b_m^\dagger|0>\mathbf{Q}_{\acute{b}ij\mu}+b_i^\dagger|0>\mathbf{Q}_{\acute{b}\mu}^i \tag{20}$$

where the Latin letters $i,j,k,l,m,...$ are $SU(5)$ indices and the Greek letters $\mu,\nu,\rho,...$ represent $SO(10)$ indices. The Latin subscripts $\acute{a},\acute{b},\acute{c},\acute{d}(=1,2,3)$ are reserved for generation indices. The reducible fields appearing in Eqs.(19) and (20) can be identified in $SU(5)$ notation as follows [4]:

$$\begin{aligned}10=5+\overline{5}:\quad &\mathbf{P}_\mu=(\mathbf{P}_{c_k},\mathbf{P}_{\overline{c}_k})\equiv\left(\mathbf{P}^k,\mathbf{P}_k\right)\\ \overline{10}=5+\overline{5}:\quad &\mathbf{Q}_\mu=(\mathbf{Q}_{c_k},\mathbf{Q}_{\overline{c}_k})\equiv\left(\mathbf{Q}^k,\mathbf{Q}_k\right)\end{aligned} \tag{21}$$

$$\begin{aligned}100=\overline{50}+50:\quad &\mathbf{P}_\mu^{ij}=\left(\mathbf{P}_{c_k}^{ij},\mathbf{P}_{\overline{c}_k}^{ij}\right)\equiv\left(\mathbf{R}^{[ij]k},\mathbf{R}_k^{[ij]}\right)\\ \overline{100}=\overline{50}+50:\quad &\mathbf{Q}_{\mu ij}=(\mathbf{Q}_{ijc_k},\mathbf{Q}_{ij\overline{c}_k})\equiv\left(\mathbf{S}_{[ij]}^k,\mathbf{S}_{[ij]k}\right)\\ 50=45+5:\ &\mathbf{R}_k^{[ij]}=\mathbf{P}_k^{ij}+\frac{1}{4}\left(\delta_k^j\widehat{\mathbf{P}}^i-\delta_k^i\widehat{\mathbf{P}}^j\right)\\ \overline{50}=\overline{40}+\overline{10}:\ &\mathbf{R}^{[ij]k}=\epsilon^{ijlmn}\mathbf{P}_{lmn}^k+\epsilon^{ijklm}\widehat{\mathbf{P}}_{lm}\\ 50=40+10:\ &\mathbf{S}_{[ij]k}=\epsilon_{ijlmn}\mathbf{Q}_k^{lmn}+\epsilon_{ijklm}\widehat{\mathbf{Q}}^{lm}\\ \overline{50}=\overline{45}+\overline{5}:\ &\mathbf{S}_{[jk]}^i=\mathbf{Q}_{jk}^i+\frac{1}{4}\left(\delta_k^i\widehat{\mathbf{Q}}_j-\delta_j^i\widehat{\mathbf{Q}}_k\right)\end{aligned} \tag{22}$$

$$\begin{aligned}&\overline{50}=25+\overline{25}:\ \mathbf{P}_{i\mu}=(\mathbf{P}_{ic_k},\mathbf{P}_{i\overline{c}_k})\equiv\left(\mathbf{R}_i^k,\mathbf{R}_{ik}\right)\\ &50=25+25:\ \mathbf{Q}_\mu^i=\left(\mathbf{Q}_{\overline{c}_k}^i,\mathbf{Q}_{c_k}^i\right)\equiv\left(\mathbf{S}_k^i,\mathbf{S}^{ik}\right)\\ &25=24+1:\ \mathbf{R}_j^i=\mathbf{P}_j^i+\frac{1}{5}\delta_j^i\widehat{\mathbf{P}},\ \overline{25}=\overline{10}+\overline{15}:\ \mathbf{R}_{ij}=\frac{1}{2}\left(\mathbf{P}_{ij}+\mathbf{P}_{ij}^{(S)}\right)\\ &25=24+1:\ \mathbf{S}_j^i=\mathbf{Q}_j^i+\frac{1}{5}\delta_j^i\widehat{\mathbf{Q}},\ 25=10+15:\ \mathbf{S}^{ij}=\frac{1}{2}\left(\mathbf{Q}^{ij}+\mathbf{Q}_{(S)}^{ij}\right)\end{aligned} \tag{23}$$

Further, $b_i^\dagger$ and $b_i^\dagger$ $(i=1,2,..,5)$ are the fermionic creation and annihilation operators and obey the anti-commutation rules[7]

$$\{b_i, b_j^\dagger\} = \delta_i^j; \quad \{b_i, b_j\} = 0; \quad \{b_i^\dagger, b_j^\dagger\} = 0 \tag{24}$$

and the $SU(5)$ singlet state $|0>$ satisfies $b_i|0>=0$. $SO(10)$ invariance requires the following constraints in general

$$\Gamma_\mu|\Psi_{(+)\mu}>=|\Psi'_{(-)}> \tag{25}$$

$$\Gamma_\mu|\Psi_{(-)\mu}>=|\Psi'_{(+)}> \tag{26}$$

where

$$\begin{aligned}|\Psi'_{(-)}>= b_1^\dagger b_2^\dagger b_3^\dagger b_4^\dagger b_5^\dagger|0>\widehat{\mathbf{P}} + \frac{1}{12}\epsilon^{ijklm} b_k^\dagger b_l^\dagger b_m^\dagger|0>\Big(\mathbf{P}_{ij} + 6\widehat{\mathbf{P}}_{ij}\Big)\\ + b_i^\dagger|0>\Big(\mathbf{P}^i + \widehat{\mathbf{P}}^i\Big)\end{aligned} \tag{27}$$

$$\begin{aligned}|\Psi'_{(+)}>= |0>\widehat{\mathbf{P}} + \frac{1}{2} b_i^\dagger b_j^\dagger|0>\Big(\mathbf{Q}^{ij} + 6\widehat{\mathbf{Q}}^{ij}\Big)\\ + \frac{1}{24}\epsilon^{ijklm} b_j^\dagger b_k^\dagger b_l^\dagger b_m^\dagger|0>\Big(\mathbf{Q}_i + \widehat{\mathbf{Q}}_i\Big)\end{aligned} \tag{28}$$

We note in passing that to get the 144 and $\overline{144}$ spinors, $|\Upsilon_{(\pm)\mu}>$, we need to impose Eq.(17). This constrain will require setting $|\Psi'_{(\pm)}>=0$ and thus require the following constraints

$$\begin{aligned}\widehat{\mathbf{P}} = 0, \quad \widehat{\mathbf{P}}^i = -\mathbf{P}^i, \quad \widehat{\mathbf{P}}_{ij} = -\frac{1}{6}\mathbf{P}_{ij}\\ \widehat{\mathbf{Q}} = 0, \quad \widehat{\mathbf{Q}}_i = -\mathbf{Q}_i, \quad \widehat{\mathbf{Q}}^{ij} = -\frac{1}{6}\mathbf{Q}^{ij}\end{aligned} \tag{29}$$

Thus we have

$$|\Upsilon_{(\pm)\mu}>=\Big(|\Psi_{(\pm)\mu}>\Big)_{constraint\ of\ Eq.(29)} \tag{30}$$

However, as we stated already we will be dealing with the full $160+\overline{160}$ multiplets.
To normalize the $SU(5)$ fields contained in the tensor, $|\Psi_{(\pm)\mu}>$, we carry out a field redefinition

$$\begin{aligned}\{1\}: \quad \widehat{\mathbf{P}} = \sqrt{5}\widehat{\mathcal{P}}, \quad \{\overline{5}\}: \quad \mathbf{P}_i = \mathcal{P}_i, \quad \{5\}: \quad \mathbf{P}^i = \mathcal{P}^i, \quad \widehat{\mathbf{P}}^i = 2\widehat{\mathcal{P}}^i\\ \{\overline{10}\}: \quad \mathbf{P}_{ij} = \sqrt{2}\mathcal{P}_{ij}, \quad \widehat{\mathbf{P}}_{ij} = \frac{1}{2\sqrt{3}}\widehat{\mathcal{P}}_{ij}, \quad \{\overline{15}\}: \quad \mathbf{P}_{ij}^{(S)} = \sqrt{2}\mathcal{P}_{ij}^{(S)}\\ \{24\}: \quad \mathbf{P}_j^i = \mathcal{P}_j^i, \quad \{\overline{40}\}: \quad \mathbf{P}_{ijk}^l = \frac{1}{6}\mathcal{P}_{ijk}^l, \quad \{45\}: \quad \mathbf{P}_k^{ij} = \mathcal{P}_k^{ij}\end{aligned} \tag{31}$$

$$\{1\}: \quad \widehat{\mathbf{Q}} = \sqrt{5}\widehat{\mathcal{Q}}, \quad \{5\}: \quad \mathbf{Q}^i = \mathcal{Q}^i, \quad \{\overline{5}\}: \quad \mathbf{Q}_i = \mathcal{Q}_i, \quad \widehat{\mathbf{Q}}_i = 2\widehat{\mathcal{Q}}_i$$
$$\{10\}: \quad \mathbf{Q}^{ij} = \sqrt{2}\mathcal{Q}_{ij}, \quad \widehat{\mathbf{Q}}^{ij} = \frac{1}{2\sqrt{3}}\widehat{\mathcal{Q}}^{ij}, \quad \{15\}: \quad \mathbf{Q}^{ij}_{(S)} = \sqrt{2}\mathcal{Q}^{ij}_{(S)}$$
$$\{24\}: \quad \mathbf{Q}^i_j = \mathcal{Q}^i_j, \quad \{40\}: \quad \mathbf{Q}^{ijk}_l = \frac{1}{6}\mathcal{Q}^{ijk}_l, \quad \{\overline{45}\}: \quad \mathbf{Q}^k_{ij} = \mathcal{Q}^k_{ij} \tag{32}$$

In terms of the normalized fields, the kinetic energy of the 160 and $\overline{160}$, i.e., $- < \partial_A \Psi_{(\pm)\mu} | \partial^A \Psi_{(\pm)\mu} >$, where A is the Lorentz index, takes the form

$$\begin{aligned}
\mathsf{L}^{\overline{160}}_{kin} = & -\partial_A \widehat{\mathcal{P}}^\dagger \partial_A \widehat{\mathcal{P}} - \partial_A \mathcal{P}^\dagger_i \partial^A \mathcal{P}_i - \partial_A \mathcal{P}^{i\dagger} \partial^A \mathcal{P}^i - \partial_A \widehat{\mathcal{P}}^{i\dagger} \partial^A \widehat{\mathcal{P}}^i \\
& -\frac{1}{2!}\partial_A \mathcal{P}^\dagger_{ij} \partial^A \mathcal{P}_{ij} - \frac{1}{2!}\partial_A \widehat{\mathcal{P}}^\dagger_{ij} \partial^A \widehat{\mathcal{P}}_{ij} - \frac{1}{2!}\partial_A \mathcal{P}^{(S)\dagger}_{ij} \partial^A \mathcal{P}^{(S)}_{ij} \\
& -\partial_A \mathcal{P}^{i\dagger}_j \partial^A \mathcal{P}^i_j - \frac{1}{3!}\partial_A \mathcal{P}^{l\dagger}_{ijk} \partial^A \mathcal{P}^l_{ijk} - \frac{1}{2!}\partial_A \mathcal{P}^{ij\dagger}_k \partial^A \mathcal{P}^{ij}_k
\end{aligned} \tag{33}$$

$$\begin{aligned}
\mathsf{L}^{160}_{kin} = & -\partial_A \widehat{\mathcal{Q}}^\dagger \partial_A \widehat{\mathcal{Q}} - \partial_A \mathcal{Q}^{i\dagger} \partial^A \mathcal{Q}^i - \partial_A \mathcal{Q}^\dagger_i \partial^A \mathcal{Q}_i - \partial_A \widehat{\mathcal{Q}}^\dagger_i \partial^A \widehat{\mathcal{Q}}_i \\
& -\frac{1}{2!}\partial_A \mathcal{Q}^{ij\dagger} \partial^A \mathcal{Q}^{ij} - \frac{1}{2!}\partial_A \widehat{\mathcal{Q}}^{ij\dagger} \partial^A \widehat{\mathcal{Q}}^{ij} - \frac{1}{2!}\partial_A \mathcal{Q}^{ij\dagger}_{(S)} \partial^A \mathcal{Q}^{ij}_{(S)} \\
& -\partial_A \mathcal{Q}^{i\dagger}_j \partial^A \mathcal{Q}^i_j - \frac{1}{3!}\partial_A \mathcal{Q}^{ijk\dagger}_l \partial^A \mathcal{Q}^{ijk}_l - \frac{1}{2!}\partial_A \mathcal{Q}^{k\dagger}_{ij} \partial^A \mathcal{Q}^k_{ij}
\end{aligned} \tag{34}$$

4 Symmetry Breaking

In this section we discuss how $SO(10)$ breaks to $SU(3)_C \times SU(2)_L \times U(1)_Y$. For this purpose we consider a superpotential of the form

$$\begin{aligned}
\mathsf{W} = & M(\overline{160}_H \times 160_H) \\
& + \frac{\lambda_1}{M'}(\overline{160}_H \times 160_H)_1 (\overline{160}_H \times 160_H)_1 \\
& + \frac{\lambda_{45}}{M'}(\overline{160}_H \times 160_H)_{45} (\overline{160}_H \times 160_H)_{45} \\
& + \frac{\lambda_{210}}{M'}(\overline{160}_H \times 160_H)_{210} (\overline{160}_H \times 160_H)_{210}
\end{aligned} \tag{35}$$

There are of course many more terms that one can add to Eq.(35) but we consider only the terms displayed in Eq.(35) for simplicity. The relevant terms in the superpotential that accomplish symmetry breaking are

$$\mathsf{W}_{_{SB}} = M\mathbf{Q}^i_\mu \mathbf{P}_{i\mu} + \alpha_{_1} \mathbf{Q}^i_\mu \mathbf{P}_{i\mu} \mathbf{Q}^j_\nu \mathbf{P}_{j\nu} + \alpha_{_2} \mathbf{Q}^i_\mu \mathbf{P}_{j\mu} \mathbf{Q}^j_\nu \mathbf{P}_{j\nu} \tag{36}$$

where

$$\alpha_1 = \frac{1}{M'}\left(-2\lambda_1 - \lambda_{45} + \frac{1}{6}\lambda_{210}\right)$$
$$\alpha_2 = -\frac{1}{M'}\left(4\lambda_{45} + \lambda_{210}\right) \tag{37}$$

Expanding into the irreducible components we find

$$W = M\mathcal{Q}^i_j\mathcal{P}^j_i + \alpha_1\mathcal{Q}^i_j\mathcal{P}^j_i\mathcal{Q}^k_l\mathcal{P}^l_k + \alpha_2\mathcal{Q}^i_k\mathcal{P}^k_j\mathcal{Q}^j_l\mathcal{P}^l_i + M\widehat{\mathcal{Q}}\widehat{\mathcal{P}} + 2\left(\alpha_1 + \frac{2}{5}\alpha_2\right)\mathcal{Q}^k_l\mathcal{P}^l_k\widehat{\mathcal{Q}}\widehat{\mathcal{P}}$$
$$+\frac{2}{\sqrt{5}}\alpha_2\mathcal{Q}^i_k\mathcal{P}^k_j\mathcal{Q}^j_i\widehat{\mathcal{P}} + \frac{2}{\sqrt{5}}\alpha_1\mathcal{Q}^i_k\mathcal{P}^k_j\mathcal{P}^j_i\widehat{\mathcal{Q}} + \frac{1}{5}\alpha_2\mathcal{Q}^k_l\mathcal{Q}^l_k\widehat{\mathcal{P}}\widehat{\mathcal{P}} + \frac{1}{5}\alpha_1\mathcal{P}^k_l\mathcal{P}^l_k\widehat{\mathcal{Q}}\widehat{\mathcal{Q}}$$
$$+\left(\alpha_1 + \frac{1}{5}\alpha_2\right)\widehat{\mathcal{Q}}\widehat{\mathcal{P}}\widehat{\mathcal{Q}}\widehat{\mathcal{P}} \tag{38}$$

In the minimization we look for solutions of the type

$$< \mathcal{Q}^i_j >= q\ diag(2,2,2,-3,-3), \quad < \mathcal{P}^i_j >= p\ diag(2,2,2,-3,-3) \tag{39}$$

One finds the following results from the minimization of the potential

$$Mp + 2p^2q\left(30\alpha_1 + 7\alpha_2\right) + 2\left(\alpha_1 + \frac{2}{5}\alpha_2\right)pQ_0P_0$$
$$+\frac{1}{75}\alpha_2 qP_0^2 + +\frac{26}{5\sqrt{5}}\alpha_2\left(pQ_0 + 2qP_0\right)p = 0 \tag{40}$$

$$Mq + 2q^2p\left(30\alpha_1 + 7\alpha_2\right) + 2\left(\alpha_1 + \frac{2}{5}\alpha_2\right)qQ_0P_0$$
$$+\frac{1}{75}\alpha_2 pQ_0^2 + +\frac{26}{5\sqrt{5}}\alpha_2\left(qP_0 + 2pQ_0\right)q = 0 \tag{41}$$

$$60\left[\left(\alpha_1 + \frac{2}{5}\alpha_2\right)qp + M\right]P_0 + \frac{2}{5}\alpha_2 p^2 Q_0 + \frac{156}{\sqrt{5}}\alpha_2 p^2 q$$
$$+2\left(\alpha_1 + \frac{1}{5}\alpha_2\right)Q_0P_0^2 = 0 \tag{42}$$

and

$$60\left[\left(\alpha_1 + \frac{2}{5}\alpha_2\right)qp + M\right]Q_0 + \frac{2}{5}\alpha_2 q^2 P_0 + \frac{156}{\sqrt{5}}\alpha_2 pq^2$$
$$+2\left(\alpha_1 + \frac{1}{5}\alpha_2\right)Q_0^2P_0 = 0 \tag{43}$$

where $Q_0 =< \widehat{\mathcal{Q}} >$ and $P_0 =< \widehat{\mathcal{P}} >$. The D-flatness condition $< 144 >=< \overline{144} >$ gives $q = p$. With the above vacuum expectation value (VEV), spontaneous breaking occurs so that $SO(10) \to SU(3)_C \times SU(2)_L \times U(1)_Y$. We note that the VEVs Q_0 and P_0 do not play a role in the above breakdown as this breakdown will occur even when $Q_0 = 0 = P_0$.

5 Higgs Phenomenon and Mass Growth

We outline here the Higgs phenomenon and the mass growth associated with the spontaneous breaking given by Eqs.(40-43). The fields that participate in the Higgs phenomenon include the 45 vector super multiplet that belongs to the adjoint representation of $SO(10)$ and the $144 + \overline{144}$ chiral superfields. For the analysis of the Higgs phenomenon it is useful to decompose the 45-plet of $SO(10)$ in multiplets of $SU(5)$ so that

$$45 = 1 + 10 + \overline{10} + 24 \tag{44}$$

After spontaneous symmetry breaking the 10_{45} massless vector super multiplet absorbs the 10_{144} chiral multiplet to become a 10_{45} massive vector super multiplet with spins $(1, \frac{1}{2}, 0)$. Similarly, the $\overline{10}_{45}$ vector super multiplet absorbs the $\overline{10}_{\overline{144}}$ chiral multiplet to become the $\overline{10}_{45}$ massive vector super multiplet. Now the 24 plet of $SU(5)$ decomposes under $SU(3)_C \times SU(2)_L$ as follows

$$24 = (8, 1) + (\bar{3}, 2) + (3, 2) + (1, 3) + (1, 1) \tag{45}$$

After spontaneous breaking the super vector multiplets with the quantum numbers $(\bar{3}, 2) + (3, 2)$ absorb one linear combination of the chiral multiplets $((\bar{3}, 2) + (3, 2))_{144}$ and $((\bar{3}, 2) + (3, 2))_{\overline{144}}$ becoming a massive $(\bar{3}, 2) + (3, 2)$ vector super multiplets while the orthogonal linear combination of $((\bar{3}, 2)+(3, 2))_{144}$ and $((\bar{3}, 2)+(3, 2))_{\overline{144}}$ which is not absorbed becomes massive. The vector super multiplets corresponding to $(8, 1)+(1, 3)+(1, 1)$ remain massless. The chiral super multiplets corresponding to $(8, 1) + (1, 3)$ become massive. (The $(1, 1)$ components of 24_{144} and $24_{\overline{144}}$ require special treatment and we return to it below). Thus we have accounted for the mass growth of the $10 + \overline{10}$ vector super multiplet and the mass growth of the 12 components $(\bar{3}, 2) + (3, 2)$ of the 24 plet vector super multiplet. This leaves us to discuss mass growth of the singlet vector super multiplet in Eq.(44). This mass growth comes about by absorption of the chiral superfield combination $(\frac{2}{5}\Sigma^a_a - \frac{3}{5}\Sigma^\alpha_\alpha)$ where the repeated indices are summed (a is the color index which takes on values 1,2,3 and α is the $SU(2)$ index and takes on values 4,5), and Σ^i_j is a linear combination of $\mathcal{Q}^i_j$ and $\mathcal{P}^i_j$. Since Σ is traceless the above equals $\Sigma^a_a = -\Sigma^\alpha_\alpha$. Thus the singlet vector super multiplet absorbs a linear combination of $\mathcal{P}^a_a$ and $\mathcal{Q}^a_a$ becoming a singlet massive vector super multiplet while the orthogonal combination of the chiral superfields $\mathcal{P}^a_a$ and $\mathcal{Q}^a_a$ becomes massive. Thus after the symmetry breaking and Higgs phenomenon only the $(1, 8) + (1, 3) + (1, 1)$ vector supermultiplet remains massless and the remaining components of 45 of the vector super

multiplet become massive. Similarly, all the unabsorbed components of the $144+\overline{144}$ become massive. At this stage the gauge group $SO(10)$ has broken down to $SU(3)_C \times SU(2)_L \times U(1)_Y$. To accomplish the breaking of the electro-weak symmetry we need a pair of Higgs doublets. Such a possibility arises for $\mathcal{Q}_\alpha$, $\mathcal{P}^\alpha$.

To exhibit this we need to compute masses for $\mathcal{Q}_i$, $\mathcal{P}^i$. It is also instructive to compute masses for $\mathcal{Q}^i$, $\mathcal{P}_i$, $\widehat{\mathcal{Q}}_i$, $\widehat{\mathcal{P}}^i$, $\mathcal{Q}^i_{jk}$ and $\mathcal{P}^{ij}_k$ The relevant terms in the superpotential are

$$\begin{aligned}
\mathsf{W}_{_{mass}} &= \left[M + \frac{1}{M'}\left(-4\lambda_1 + 6\lambda_{45} - \frac{1}{3}\lambda_{210}\right) < \mathbf{S}^m_n \mathbf{R}^n_m >\right]\left(\mathcal{Q}_i\mathcal{P}^i + \mathcal{Q}^i\mathcal{P}_i\right) \\
&- \left[\frac{8}{3}\frac{\lambda_{210}}{M'} < \mathbf{S}^i_m \mathbf{R}^m_j >\right]\mathcal{Q}_i\mathcal{P}^j + \left[\frac{1}{M'}\left(8\lambda_{45} - \frac{2}{3}\lambda_{210}\right) < \mathbf{S}^i_m \mathbf{R}^m_j >\right]\mathbf{S}^l_{[in]}\mathbf{R}^{[nj]}_l \\
&+ \left[-\frac{1}{2}M + \frac{1}{M'}\left(2\lambda_1 + \lambda_{45} - \frac{1}{6}\lambda_{210}\right) < \mathbf{S}^m_n \mathbf{R}^n_m >\right]\mathbf{S}^k_{[ij]}\mathbf{R}^{[ij]}_k
\end{aligned} \tag{46}$$

Eq.(46) makes it apparent why for technical reasons we need to keep the $160+\overline{160}$ multiplet. To see this let us set all the couplings λ to zero in Eq.(46) so that the only terms surviving are proportional to M. Next suppose we impose on $\mathbf{S}^k_{[ij]}$ and $\mathbf{R}^{[ij]}_k$ the constraint of Eq.(29) so that we are strictly considering only the $144+\overline{144}$ multiplet. Then we see that the last line of Eq.(46) contributes an additional mass term $-\frac{M}{4}\mathcal{Q}_i\mathcal{P}^i$ while there is no such term for $\mathcal{Q}^i\mathcal{P}_i$. This additional term is clearly not desired and the reason for its appearance is that we are identifying the 5 plet in $\mathbf{R}^{[ij]}_k$ with $\mathcal{P}^i$ and $\bar{5}$ plet in $\mathbf{S}^k_{[ij]}$ with $\mathcal{Q}_i$ because of Eq.(29) which feeds in the undesired additional term. Thus for book keeping we must not impose the constraint of Eq.(29) in the beginning. Returning to Eq.(46), the corresponding mass terms in the Lagrangian are given by

$$\mathsf{L} = \left|\frac{\partial \mathsf{W}_{_{mass}}}{\partial \mathcal{Q}_i}\right|^2 + \left|\frac{\partial \mathsf{W}_{_{mass}}}{\partial \mathcal{P}^i}\right|^2 + \left|\frac{\partial \mathsf{W}_{_{mass}}}{\partial \mathcal{Q}^i}\right|^2 + \left|\frac{\partial \mathsf{W}_{_{mass}}}{\partial \mathcal{P}_i}\right|^2 + \left|\frac{\partial \mathsf{W}_{_{mass}}}{\partial \mathbf{S}^k_{[ij]}}\right|^2 + \left|\frac{\partial \mathsf{W}_{_{mass}}}{\partial \mathbf{R}^{[ij]}_k}\right|^2 \tag{47}$$

Explicitly we have

$$\begin{aligned}
\mathsf{L} &= |\sigma|^2\left(\mathcal{P}_i\mathcal{P}^\dagger_i + \mathcal{Q}^i\mathcal{Q}^{i\dagger}\right) \\
&+ |\sigma + \omega_{_1}\beta|^2\left(\mathcal{P}^\alpha\mathcal{P}^{\alpha\dagger} + \mathcal{Q}_\alpha\mathcal{Q}^\dagger_\alpha\right) \\
&+ |\sigma + \omega_{_2}\beta|^2\left(\mathcal{P}^a\mathcal{P}^{a\dagger} + \mathcal{Q}_a\mathcal{Q}^\dagger_a\right) \\
+\frac{1}{2}\left[|\rho - \omega_{_1}\gamma|^2 + 3\left|\rho - \frac{1}{2}\left(\omega_{_1} + \omega_{_2}\right)\gamma\right|^2\right] &\left(\widehat{\mathcal{P}}^\alpha\widehat{\mathcal{P}}^{\alpha\dagger} + \widehat{\mathcal{Q}}_\alpha\widehat{\mathcal{Q}}^\dagger_\alpha\right) \\
+\left[|\rho - \omega_{_1}\gamma|^2 + \left|\rho - \frac{1}{2}\left(\omega_{_1} + \omega_{_2}\right)\gamma\right|^2\right] &\left(\widehat{\mathcal{P}}^a\widehat{\mathcal{P}}^{a\dagger} + \widehat{\mathcal{Q}}_a\widehat{\mathcal{Q}}^\dagger_a\right)
\end{aligned}$$

$$
\begin{aligned}
&+|\rho-\omega_1\gamma|^2\left(\mathcal{P}_k^{\alpha\beta}\mathcal{P}_k^{\alpha\beta\dagger}+\mathcal{Q}_{\alpha\beta}^{k}\mathcal{Q}_{\alpha\beta}^{k\dagger}\right)\\
&+|\rho-\omega_2\gamma|^2\left(\mathcal{P}_k^{ab}\mathcal{P}_k^{ab\dagger}+\mathcal{Q}_{ab}^{k}\mathcal{Q}_{ab}^{k\dagger}\right)\\
&+2\left|\rho-\frac{1}{2}\left(\omega_1+\omega_2\right)\gamma\right|^2\left(\mathcal{P}_k^{a\alpha}\mathcal{P}_k^{a\alpha\dagger}+\mathcal{Q}_{a\alpha}^{k}\mathcal{Q}_{a\alpha}^{k\dagger}\right)\\
&+\left[\left|\rho-\frac{1}{2}\left(\omega_1+\omega_2\right)\gamma\right|^2-|\rho-\omega_1\gamma|^2\right]\left(\mathcal{P}_\alpha^{a\alpha}\widehat{\mathcal{P}}^{a\dagger}+\mathcal{Q}_{a\alpha}^{\alpha}\widehat{\mathcal{Q}}_a^{\dagger}+H.C.\right)\\
&+\left[\left|\rho-\frac{1}{2}\left(\omega_1+\omega_2\right)\gamma\right|^2-|\rho-\omega_2\gamma|^2\right]\left(\mathcal{P}_a^{\alpha a}\widehat{\mathcal{P}}^{\alpha\dagger}+\mathcal{Q}_{\alpha a}^{a}\widehat{\mathcal{Q}}_\alpha^{\dagger}+H.C.\right)
\end{aligned}
\tag{48}
$$

where

$$
\begin{aligned}
\sigma &= M+\frac{qp}{M'}\left(-4\lambda_1+6\lambda_{45}-\frac{1}{3}\lambda_{210}\right)\left(30+\frac{P_0Q_0}{pq}\right)\\
\rho &= -\frac{1}{2}M+\frac{qp}{M'}\left(2\lambda_1+\lambda_{45}-\frac{1}{6}\lambda_{210}\right)\left(30+\frac{P_0Q_0}{pq}\right)\\
\omega_1 &= qp\left[9-\frac{3}{\sqrt{5}}\left(\frac{P_0}{p}+\frac{Q_0}{q}\right)+\frac{1}{5}\frac{P_0Q_0}{pq}\right]\\
\omega_2 &= qp\left[4+\frac{2}{\sqrt{5}}\left(\frac{P_0}{p}+\frac{Q_0}{q}\right)+\frac{1}{5}\frac{P_0Q_0}{pq}\right]\\
\gamma &= \frac{1}{M'}\left(8\lambda_{45}-\frac{2}{3}\lambda_{210}\right)\\
\beta &= -\frac{8}{3}\frac{\lambda_{210}}{M'}
\end{aligned}
\tag{49}
$$

The masses for the fields $\mathcal{P}^i$, $\mathcal{Q}_i$, $\mathcal{Q}^i, \mathcal{P}_i$, $\mathcal{Q}^k_{ij}$, and $\mathcal{P}^{ij}_k$ can be computed from Eqs.(40-43) and (48). It is easily checked that the masses vanish unless one has a non-vanishing λ_{45} or λ_{210}. A scrutiny of the mass growth above shows that the masses of the Higgs doublets $\mathcal{Q}_\alpha, \mathcal{P}^\alpha$ are split from the Higgs triplets $\mathcal{Q}_a, \mathcal{P}^a$. Indeed this allows one to fine tune the masses of the Higgs doublets to zero by the condition

$$
\begin{aligned}
&\frac{MM'}{qp}+(-120\lambda_1+180\lambda_{45})\left(1+\frac{1}{30}\frac{P_0Q_0}{pq}\right)\\
&+\lambda_{210}\left[-34+\frac{8}{\sqrt{5}}\left(\frac{P_0}{p}+\frac{Q_0}{q}\right)-\frac{13}{15}\frac{P_0Q_0}{pq}\right]=0
\end{aligned}
\tag{50}
$$

With this constraint one finds that the Higgs doublets $\mathcal{Q}_\alpha, \mathcal{P}^\alpha$ are massless while the Higgs triplets $\mathcal{Q}_a, \mathcal{P}^a$ are massive. Thus the Higgs triplet mass M_{H_3} of the fields $\mathcal{Q}_a, \mathcal{P}^a$, under the constraint that the Higgs doublet $\mathcal{Q}_\alpha, \mathcal{P}^\alpha$ be massless, is given by

$$
M_{H_3}=\frac{40}{3}\frac{qp}{M'}\left[1-\frac{1}{\sqrt{5}}\left(\frac{P_0}{p}+\frac{Q_0}{q}\right)\right]\lambda_{210}
\tag{51}
$$

We note that it is not possible to achieve a doublet-triplet splitting for the multiplets $\mathcal{Q}^i$ and $\mathcal{P}_i$. Thus the doublet-triplet splitting we are considering is unique. Further, we find that $\mathcal{Q}^i$, $\mathcal{P}_i$ develop a mass

$$M_{\mathcal{Q}^i,\mathcal{P}_i} = 24\frac{qp}{M'}\left[1-\frac{1}{3\sqrt{5}}\left(\frac{P_0}{p}+\frac{Q_0}{q}\right)+\frac{1}{45}\frac{P_0Q_0}{pq}\right]\lambda_{210} \tag{52}$$

In the above Q_0 and P_0 are crucial in getting a pair of light Higgs doublets. Thus suppose we have $Q_0 = 0 = P_0$ in Eqs.(40-43). In this case one finds an additional constraint. Thus from Eqs.(42) and (43) one finds that $Q_0 = 0 = P_0$ imply that $\alpha_2 = 0$. Under this constraint Eq.(50) is not consistent with Eqs.(40) and (41). i.e., one cannot find a pair of light Higgs doublets. We note that this result is a consequence of considering only a limited number of couplings in Eq.(35). Inclusion of a larger set of couplings should allow one to get consistent solutions without inclusion of the singlet VEVs.

To summarize the results thus far we have here a complete breaking of $SO(10)$ to $SU(3)_C \times SU(2)_L \times U(1)_Y$. To get a pair of light Higgs we need to invoke a string landscape scenario which has been extensively discussed recently[9, 10]. Effectively in this framework we use a fine tuning to keep one pair of Higgs doublets light while keeping the Higgs triplets heavy. With the usual radiative electroweak symmetry breaking mechanism, the light doublets of Higgs can develop VEVs breaking the $SU(2)_L \times U(1)_Y$ symmetry down to $U(1)_{em}$. Thus with the above mechanism one can break $SO(10)$ down to $SU(3)_C \times U(1)_{em}$ with just one pair of $144 + \overline{144}$ of Higgs. A similar analysis shows that one gets masses for the remaining parts of the $144 + \overline{144}$ chiral multiplets not absorbed by the vector bosons which become heavy. The heavy spectrum of this model differs significantly from the standard $SU(5)$ and the standard $SO(10)$ models. It consists of several parts: (i) the super heavy lepto-quarks associated with the breaking of the $SO(10) \to SU(3)_C \times SU(2)_L \times U(1)_Y$, (ii) the super heavy Higgs triplet field, (iii) the components of 24 plet fields $\mathcal{P}^i_j$, $\mathcal{Q}^i_j$ which are unabsorbed by the Higgs phenomenon and become super heavy, (iv) the remaining components of $144 + \overline{144}$, aside from a 24 plets of $SU(5)$ each in 144 and $\overline{144}$ discussed in (iii) and excluding the Higgs doublets $\mathcal{Q}_\alpha, \mathcal{P}^\alpha$, which become super heavy, and (v) $16 + \overline{16}$ plet of super heavy fields. Gauge coupling unification will require a careful analysis of contribution of each of the above. We do not address this question further here.

We discuss briefly the issue of split vs non-split supersymmetry breaking. Eq.(50) is a tree level relation and its imposition to achive light Higgs doublets presumes that the loop corrections to the Higgs masses are small. Specifically it requires that the scale of supersymmetry breaking is not high, e.g., the masses of of squarks are at the electroweak scale

and not at the GUT scale, An alternative possibility is that one may impose Eq.(50) but with loop correction included. In this case we can allow for split supersymmetry scenario where the masses of the squarks are superheavy while gaugino masses may lie at the electroweak scale. Thus the model we are considering can accommodate a split or a non-split supersymmetry breaking scenario.

6 Couplings of Quarks and Leptons with 144 and $\overline{144}$ of Higgs

The 144 and $\overline{144}$ plets of Higgs have no $SO(10)$ invariant trilinear coupling with the 16 plet of matter. However one can write quartic couplings of the type $(1/M_P)(16\times 16)(144\times 144)$ and $(1/M_P)(16\times 16)(\overline{144}\times\overline{144})$, where M_P is a super heavy mass. These interactions will generate effective cubic couplings with the light Higgs doublets $\mathcal{Q}_\alpha, \mathcal{P}^\alpha$ after spontaneous breaking of $SO(10)$ discussed in Secs.3 and 4. The size of these couplings is $O(M_G/M_P)$. The most general set of quartic couplings involving quarks and leptons are[5]

$$\begin{aligned}
&(16\times 16)_{10}(144\times 144)_{10}, \quad (16\times 16)_{10}(\overline{144}\times\overline{144})_{10},\\
&(16\times 16)_{\overline{126}}(144\times 144)_{126}, \quad (16\times 16)_{\overline{126}}(\overline{144}\times\overline{144})_{126}
\end{aligned} \tag{53}$$

The couplings $(16\times 16)_{10}\,(144\times 144)_{10}$ and $(16\times 16)_{10}\left(\overline{144}\times\overline{144}\right)_{10}$ arise from the following structures

$$\begin{aligned}
\mathsf{W}^{(10)} &= \frac{1}{2}\Phi_{\nu\mathcal{U}}\mathcal{M}^{(10)}_{\mathcal{U}\mathcal{U}'}\Phi_{\nu\mathcal{U}'} + h^{(10)}_{\acute{a}\acute{b}} < \Upsilon^*_{(+)\acute{a}\mu}|B\Gamma_\nu|\Upsilon_{(+)\acute{b}\mu} > k^{(10)}_{\mathcal{U}}\Phi_{\nu\mathcal{U}}\\
&+ f^{(10)}_{\acute{a}\acute{b}} < \Psi^*_{(+)\acute{a}}|B\Gamma_\nu|\Psi_{(+)\acute{b}} > l^{(10)}_{\mathcal{U}}\Phi_{\nu\mathcal{U}} + +\bar{h}^{(10)}_{\acute{a}\acute{b}} < \Upsilon^*_{(-)\acute{a}\mu}|B\Gamma_\nu|\Upsilon_{(-)\acute{b}\mu} > \bar{k}^{(10)}_{\mathcal{U}}\Phi_{\nu\mathcal{U}}
\end{aligned} \tag{54}$$

where the indices $\mathcal{U}$,$\mathcal{U}'$ run over several Higgs representations of the same kind, $\mathcal{M}^{(10)}$ represents the mass matrix and $f^{(10)}$, $k^{(10)}$, $\bar{k}^{(10)}$, and $l^{(10)}$ are constants. B is the usual $SO(10)$ charge conjugation operator defined by $B=\prod_{\mu=odd}\Gamma_\mu$. The semi-spinors $\Psi_{(\pm)}$ transforms as a $16(\overline{16})$-dimensional irreducible representation of $SO(10)$ and contain $1+\overline{5}+10(1+5+\overline{10})$ in its $SU(5)$ decomposition. $|\Psi_{(+)\acute{a}}>$ is given by

$$|\Psi_{(+)\acute{a}}>=|0>\mathbf{M}_{\acute{a}} + \frac{1}{2}b_i^\dagger b_j^\dagger|0>\mathbf{M}^{ij}_{\acute{a}} + \frac{1}{24}\epsilon^{ijklm}b_j^\dagger b_k^\dagger b_l^\dagger b_m^\dagger|0>\mathbf{M}_{\acute{a}i} \tag{55}$$

[5]We have not included the $(16\times 16)_{120}(144\times 144)_{120}$ and $(16\times 16)_{120}(\overline{144}\times\overline{144})_{120}$ couplings here since these couplings are anti-symmetric in the generation indices and vanish with only a single $144+\overline{144}$.

Elimination of the super heavy fields $\Phi_{\nu\mathcal{U}}$ using the F-flatness condition gives

$$\mathsf{W}^{(10)}_{dim-5} = \mathsf{W}^{(16\times16)_{10}(144\times144)_{10}} + \mathsf{W}^{(16\times16)_{10}(\overline{144}\times\overline{144})_{10}} \tag{56}$$

where

$$\begin{aligned}
\mathsf{W}^{(16\times16)_{10}(\overline{144}\times\overline{144})_{10}} &= -2\xi^{(10)}_{\acute{a}\acute{b},\acute{c}\acute{d}} < \Psi^*_{(+)\acute{a}}|B\Gamma_\rho|\Psi_{(+)\acute{b}} >< \Upsilon^*_{(+)\acute{c}\nu}|\Gamma_\rho|\Upsilon_{(+)\acute{d}\nu} > \\
&= -4\xi^{(10)}_{\acute{a}\acute{b},\acute{c}\acute{d}}[< \Psi^*_{(+)\acute{a}}|Bb_i|\Psi_{(+)\acute{b}} >< \Upsilon^*_{(+)\acute{c}\nu}|Bb_i^\dagger|\Upsilon_{(+)\acute{d}\nu} > \\
&\quad + < \Psi^*_{(+)\acute{a}}|Bb_i^\dagger|\Psi_{(+)\acute{b}} >< \Upsilon^*_{(+)\acute{c}\nu}|Bb_i|\Upsilon_{(+)\acute{d}\nu} >] \\
&= 2\xi^{(10)(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}[\epsilon_{jklmn}\mathbf{M}^{\mathbf{T}}_{\acute{a}i}\mathbf{M}^{ij}_{\acute{b}}\mathbf{P}^{kl\mathbf{T}}_{\acute{c}\mu}\mathbf{P}^{mn}_{\acute{d}\mu} - 8\mathbf{M}^{\mathbf{T}}_{\acute{a}i}\mathbf{M}^{ij}_{\acute{b}}\mathbf{P}^{\mathbf{T}}_{\acute{c}j\mu}\mathbf{P}_{\acute{d}\mu} \\
&\quad +\epsilon_{jklmn}\mathbf{M}^{kl\mathbf{T}}_{\acute{a}}\mathbf{M}^{mn}_{\acute{b}}\mathbf{P}^{\mathbf{T}}_{\acute{c}i\mu}\mathbf{P}^{ij}_{\acute{d}\mu} - 8\mathbf{M}^{\mathbf{T}}_{\acute{a}j}\mathbf{M}_{\acute{b}}\mathbf{P}^{\mathbf{T}}_{\acute{c}i\mu}\mathbf{P}^{ij}_{\acute{d}\mu}]
\end{aligned} \tag{57}$$

and

$$\begin{aligned}
\mathsf{W}^{(16\times16)_{10}(144\times144)_{10}} &= -2\zeta^{(10)}_{\acute{a}\acute{b},\acute{c}\acute{d}} < \Psi^*_{(+)\acute{a}}|B\Gamma_\rho|\Psi_{(+)\acute{b}} >< \Upsilon^*_{(-)\acute{c}\nu}|B\Gamma_\rho|\Upsilon_{(-)\acute{d}\nu} > \\
&= -4\zeta^{(10)}_{\acute{a}\acute{b},\acute{c}\acute{d}}[< \Psi^*_{(+)\acute{a}}|Bb_i|\Psi_{(+)\acute{b}} >< \Upsilon^*_{(-)\acute{c}\nu}|Bb_i^\dagger|\Upsilon_{(-)\acute{d}\nu} > \\
&\quad + < \Psi^*_{(+)\acute{a}}|Bb_i^\dagger|\Psi_{(+)\acute{b}} >< \Upsilon^*_{(-)\acute{c}\nu}|Bb_i|\Upsilon_{(-)\acute{d}\nu} >] \\
&= 2\zeta^{(10)(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}[8\mathbf{M}^{\mathbf{T}}_{\acute{a}i}\mathbf{M}^{ij}_{\acute{b}}\mathbf{Q}^{k\mathbf{T}}_{\acute{c}\mu}\mathbf{Q}_{\acute{d}kj\mu} - 8\mathbf{M}^{\mathbf{T}}_{\acute{a}i}\mathbf{M}_{\acute{b}}\mathbf{Q}^{i\mathbf{T}}_{\acute{c}\mu}\mathbf{Q}_{\acute{d}\mu} \\
&\quad -\mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}^{kl}_{\acute{b}}\mathbf{Q}^{\mathbf{T}}_{\acute{c}kl\mu}\mathbf{Q}_{\acute{d}ij\mu} + \mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}^{kl}_{\acute{b}}\mathbf{Q}^{\mathbf{T}}_{\acute{c}ik\mu}\mathbf{Q}_{\acute{d}jl\mu} \\
&\quad -\mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}^{kl}_{\acute{b}}\mathbf{Q}^{\mathbf{T}}_{\acute{c}il\mu}\mathbf{Q}_{\acute{d}jk\mu} + \epsilon^{ijklm}\mathbf{M}^{\mathbf{T}}_{\acute{a}i}\mathbf{M}_{\acute{b}}\mathbf{Q}^{\mathbf{T}}_{\acute{c}jk\mu}\mathbf{Q}_{\acute{d}lm\mu} \\
&\quad +\epsilon_{ijklm}\mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}^{kl}_{\acute{b}}\mathbf{Q}^{m\mathbf{T}}_{\acute{c}\mu}\mathbf{Q}_{\acute{d}\mu}]
\end{aligned} \tag{58}$$

and where

$$\begin{aligned}
\xi^{(10)}_{\acute{a}\acute{b},\acute{c}\acute{d}} &= f^{(10)}_{\acute{a}\acute{b}}h^{(10)}_{\acute{c}\acute{d}}l^{(10)}_{\mathcal{U}}\widetilde{\mathcal{M}}^{(10)}_{\mathcal{U}\mathcal{U}'}k^{(10)}_{\mathcal{U}'} \\
\zeta^{(10)}_{\acute{a}\acute{b},\acute{c}\acute{d}} &= f^{(10)}_{\acute{a}\acute{b}}\bar{h}^{(10)}_{\acute{c}\acute{d}}l^{(10)}_{\mathcal{U}}\widetilde{\mathcal{M}}^{(10)}_{\mathcal{U}\mathcal{U}'}\bar{k}^{(10)}_{\mathcal{U}'} \\
\widetilde{\mathcal{M}}^{(10)} &= \left[\mathcal{M}^{(10)} + \left(\mathcal{M}^{(10)}\right)^{\mathbf{T}}\right]^{-1}
\end{aligned} \tag{59}$$

We note that $\xi^{(10)(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}$, and $\zeta^{(10)(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}$, are the same as defined by Eq.(59), provided we replace h's, $\bar{h}$'s, f's by $h^{(+)}$'s, $\bar{h}^{(+)}$'s, $f^{(+)}$'s respectively where $(+)$ indicates that the couplings are symmetric, i.e., $h^{(10)(\pm)}_{\acute{a}\acute{b}} = \frac{1}{2}\left(h^{(10)}_{\acute{a}\acute{b}} \pm h^{(10)}_{\acute{b}\acute{a}}\right)$. The above couplings produce quark and lepton masses after GUT symmetry breaking followed by spontaneous breaking of the electroweak symmetry. A preliminary analysis shows that the relation on Yukawa couplings such as $h_b = h_t$ does not hold. It would be interesting to study the quark-lepton textures[11, 12, 13] in this framework. Further, the preliminary analysis shows that baryon and lepton number

violating dimension five operators in this theory are rather different than what one has in the usual $SU(5)$ and $SO(10)$ models[14, 15, 16, 17, 18]. Thus, for example, in $SU(5)$ unified models the baryon and lepton number violating dimension five operators arise from the Higgs triplet exchange from pairs of $5+\bar{5}$. In the present model there are several sources of baryon and lepton number violations, including Higgs triplets from 5 and $\bar{5}$ and from the exchange of $45+\overline{45}$ present in $144+\overline{144}$ of Higgs. A full analysis of this issue involves additional couplings where the mediation occurs via 120, 126 etc and is beyond the scope of this paper. An analysis of this will be given elsewhere[19]. We note here that if in Eq. (53) only couplings involving $1\bar{4}4$ are kept, the resulting fermion mass matrices will have the same structure as in [3] with a single 10 and one $\overline{126}$ coupling to fermions. Such matrices are fully consistent with experimental data.

7 Conclusion

In conclusion we have investigated a new class of $SO(10)$ models where the breaking of $SO(10)$ down to $SU(3)_C \times SU(2)_L \times U(1)_Y$ can occur in a single step. Further, it is possible to achieve with fine tuning, a pair of light Higgs doublets which is justifiable within a string based landscaped scenario. The light Higgs doublets allow one to break the electroweak symmetry $SU(2)_L \times U(1)_Y$ down to $U(1)_{em}$ and thus $SO(10)$ can break to $SU(3)_C \times U(1)_{em}$. The cubic interactions of the light Higgs doublets with quarks and leptons arise from quartic interactions of the type (16.16)(144.144) and $(16.16)(\overline{144}.\overline{144})$ after spontaneous breaking of $SO(10)$. In this scenario the baryon and lepton number violating dimension five operators receive contributions not just from the conventional Higgs triplet fields but also from the exchange of $45+\overline{45}$ components of $144+\overline{144}$. The above feature distinguishes the above scenario from the conventional models and would lead to different estimates on the proton lifetime and in conventional GUTs. A more detailed analysis of the quark -lepton masses as well as of the proton life time is outside the scope of this paper and will be dealt with elsewhere[19]. Finally we note that above the GUT scale one has a large number of degrees of freedom and the renormalization group evolution is very rapid and thus the theory becomes nonperturbative. We view this theory as descending directly from a theory of quantum gravity where the scale of quantum gravity (e.g., string scale) may lie close to the GUT scale.

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